## The Structure of Cool Accretion Disc in Semidetached Binaries

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#### Abstract

We present the results of qualitative consideration of possible changes occurring during the transition from the hot accretion disc to the cool one. We argue the possible existence of one more type of spiral density waves in the inner part of the disc where gasdynamical perturbations are negligible. The mechanism of formation of such a wave as well as its parameters are considered.

We also present the results of 3D gasdynamical simulation of cool accretion discs. These results confirm the hypothesis of possible formation of the spiral wave of a new, "precessional" type in the inner regions of the disc. Possible observational manifestations of this wave are discussed.

### 1 Introduction

The analysis of principal processes of matter heating and cooling in accretion discs presented in the work [1] has shown that for realistic parameters of accretion discs in semidetached binaries ( $\dot{M} \simeq 10^{-12} \div 10^{-7} M_{\odot}$ /year and  $\alpha \simeq 10^{-1} \div 10^{-2}$ )<sup>1</sup> the gas temperature in outer part of the disc lies in the

 $<sup>^1</sup>M$  – mass transfer rate;  $\alpha$  – dimensionless parameter introduced by Shakura and Sunyaev [2, 3] for expression of viscosity coefficient  $\nu = \alpha c_s H$  (H – disc semithickness,  $c_s$  – sound speed).

range from  $\sim 10^4$  K to  $\sim 10^6$  K. Earlier we have conducted 3D gasdynamical simulation of accretion discs both for a 'hot' case (the gas temperature in the outer part of the disc was 200–500 thousands K [4–14]), and for the 'cool' case (the temperature of gas in the outer part of the disc didn't exceed  $\sim 2 \times 10^4$  K [1]). The analysis of these results has shown that for both cases (i.e. independently on the disc temperature) the self-consistent solution didn't involve the shock interaction between the stream of matter from the inner Lagrangian point  $L_1$  and formed accretion disc ("hot spot"). Energy release zone ("hot line") is located outside the disc and is formed as the result of the interaction of the circumdisc halo and circumbinary envelope with the stream. The "hot line" model is found to be in a good agreement with observations [15–18].

For 'hot' solutions we have investigated both general morphology of gaseous flows in semidetached binaries and the structure of formed hot accretion discs (see, e.g., [14,19]). In particular, we have found only one arm of spiral shock wave generated by the tidal influence of the mass-losing star. The two-armed spiral shocks were discovered by Matsuda et al. in [20–22]. Nevertheless, our 3D gasdynamical simulations for the 'hot' case have shown the presence of only one-armed spiral shock while in the place where the second arm should be the stream from  $L_1$  dominates and presumably prevents the formation of the second arm of tidally induced spiral shock. Besides we have found out that for the 'hot' case the variation of mass transfer rate leads to the disc perturbations and to the formation of spiral-vortex structure in the disc [23–25].

Even a glance at the morphology of gaseous flows for the 'cool' case (see, e.g., [1,26]) discovers that the accretion disc for this case is characterized by principally different parameters as compared to the 'hot' case. In particular, the disc has more circular form and the second arm of the tidal spiral wave is present. Our analysis [1] shows that opposite to the 'hot' case tidal spiral waves don't propagate to the inner part of the cool accretion disc and are located in the outer part of the disc only.

The aim of this work is the investigation of the structure of cool accretion discs in semidetached binaries. Section 2 contains the qualitative analysis of changes occurring in transition from a hot accretion disc to the cool one. In particular, here we suggest the possible existence of one more type of spiral density waves in the inner part of the disc where gasdynamical perturbations are negligible. We also consider the mechanism of formation of such a wave and its parameters. Section 3 contains the results of 3D gasdynamical simulation of flow structure for the case when radiative cooling is effective and the gas temperature drops down to  $\sim 10^4$  K in the whole region. These results confirm the hypothesis of possible formation of a spiral wave of a new

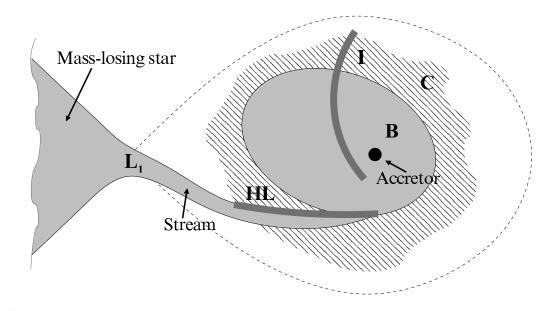


Figure 1: The sketch of main peculiarities of the morphology of gaseous flows in semidetached binaries for the case of high gas temperature.

type in the inner regions of the disc. Possible observational manifestations of the discovered spiral wave of the new, "precessional" type as well as main conclusions are drawn in Section 4.

### 2 The structure of the cool accretion disc. Qualitative consideration.

The sketch of main peculiarities of the morphology of gaseous flows in semidetached binaries for the 'hot' case is given in Fig. 1. This scheme is based on the results of 3D gasdynamical simulations published in [4–14]. In Fig. 1 the fragment of mass-losing star that fills its Roche lobe, the location of the inner Lagrangian point  $L_1$ , the stream of matter from  $L_1$ , as well as the location of the accretor are shown. A dashed line marks the Roche lobe. Following the definitions given in [11] the morphology of gaseous flows in semidetached binaries is governed by the stream of matter from  $L_1$ , quasi-elliptical accretion disc, circumdisc halo and circumbinary envelope. This classification of the main constituents is based on their physical properties: (i) if the motion of a gas particle is not determined by the gravitational field of accretor then this particle belongs to the circumbinary envelope filling the space between the components of binary; (ii) if a gas particle revolves around the accretor

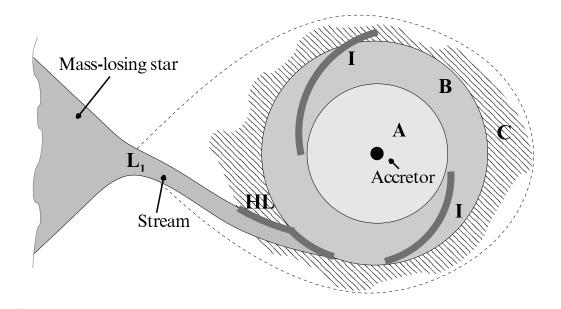


Figure 2: The sketch of main peculiarities of the morphology of gaseous flows in semidetached binaries for the case of low gas temperature.

and after that mixes with the matter of the stream then it doesn't belong to the accretion disc but forms the circumdisc halo (zone 'C' in Fig. 1); (iii) the accretion disc is formed by the matter of the stream which is gravitationally captured by the accretor and hereinafter doesn't interact with the stream but moves to the accretor losing the angular momentum (zone 'B' in Fig. 1). The interaction of matter of circumdisc halo and circumbinary envelope with the stream results in the formation of the shock located along the edge of the stream. This shock is referred as "hot line" and is marked by 'HL' in Fig. 1. Tidal action of mass-losing star results in formation of spiral shock marked by 'I' in Fig. 1. Our 3D gasdynamical simulations for the 'hot' case have shown only one-armed spiral shock while in the place where the second arm should be the flow structure is determined by the stream from  $L_1$ . It also should be stressed that the spiral shock deeply penetrates to the inner part of the disc in this case.

Let us consider the changes occurring during the transition from the hot accretion disc to the cool one. The sketch of main peculiarities of the morphology of gaseous flows in semidetached binaries for the 'cool' case when non-adiabatic processes of radiative heating and cooling result in dropping of gas temperature is given in Fig. 2. Our 3D gasdynamical simulations presented in [1] have shown that for the 'cool' case when the radiative cooling decreases the gas temperature to  $\sim 10^4$  K the solution has the same qualita-

tive features as that for the 'hot' case, namely: the interaction between the stream and the disc is shockless, the energy release zone – shock wave 'HL' – is due to the interaction with circumdisc halo and located outside the disc, being rather elongated this shock wave can also be referred as "hot line". At the same time, in the 'cool' case the accretion disc (zones 'A' and 'B' in Fig. 2) is significantly more dense as compared to the matter of the stream, the disc is thinner and has not quasi-elliptical but circular form. The size of circumdisc halo is less as well. The second arm of the tidal spiral shock is formed, the both arms don't reach the accretor but are located in the outer part of the disc. Taking into account that the stream acts on the dense inner part of the disc weakly as well as that all the shocks ("hot line" and two arms of tidal wave) are located in the outer part of the disc we can introduce a new element of flow structure for the 'cool' case: the inner region of accretion disc (zone 'A' in Fig. 2) where the influence of gasdynamical perturbations mentioned above is negligible.

Let us consider the flow of matter in the inner parts of the disc that are not subjected by gasdynamical perturbations. In the absence of external action a gas particle should revolve around the gravitation center (accretor) along the elliptical orbit. In our gasdynamical solutions (see Section 3) gas particles move along the near-circular but elliptical orbits, the accretor being located in one of the ellipse focal points. It is known (see, e.g., [27, 28]), that the influence of companion star results in retrograde precession of particle's orbit, the precession rate is proportional to orbit radius in accordance to

$$\frac{P_{pr}}{P_{orb}} \simeq \frac{4}{3} \frac{\sqrt{1+q}}{q} \left(\frac{r}{A}\right)^{-3/2} , \qquad (1)$$

where  $P_{pr}$  – period of orbit precession;  $P_{orb}$  – orbital period of binary; q – components' mass ratio; r – orbit radius; A – binary separation.

The accretion disc is formed by a multitude of particles, each of them moving along its own elliptical orbit. Due to interaction between particles the disc should be considered in gasdynamical approximation, so we should consider flowlines instead of orbits, the first ones are also being elliptical. It is known that flowlines can't intersect and can only be tangent to each other. It is also evident from geometrical consideration that in order to build the disc from non-intersecting ellipses we should embed it into each other. For the case of zero eccentricity we will obtain the circular disc. For the case of non-zero eccentricity of flowlines we can build the equilibrium solution with aligned semimajor axes of ellipses. In the presence of an external action (as it occurs in binaries) orbits begin to precess, the precession of flowlines more distant from the accretor being faster so it will overtake the flowlines with less semimajor axes. Due to impossibility of intersections between flowlines,

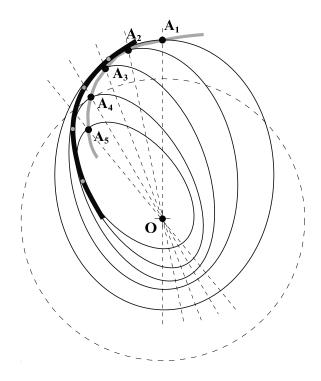


Figure 3: The scheme of generation of spiral structures in the cool disc's inner part where gasdynamical perturbations are negligible. Apastrons for each flowline are shown by black circles and marked by  $A_1, A_2, ..., A_5$ . The places of maximal attachment of flowlines are shown by gray circles.

the solution goes to equilibrium state and all flowlines tend to precess with the same angular velocity, i.e. in rigid-body mode. In this state a remote orbit should be turned at larger angle in the direction opposite to that of the matter rotation in the disc since the precession is retrograde. The precession rate value lies in the range from the one for outer ('fast') orbits and the precession rate for inner ('slow') orbits. The inner orbits are defined as those with the negligible gravitational influence of companion star as compared to the gravitational influence of accretor. The outer orbits are defined as those lying in the region without gasdynamical perturbations since latter can violate the regularity of precession. The location of both innermost and outermost orbits obviously depends on parameters of binary and parameters of mass transfer (e.g., mass transfer rate) so we can expect the various values of precession rate for different systems.

Let us consider a solution with the semimajor axis misaligned w.r.t. some chosen direction and the angle of misalignment (turn angle) being proportional to the value of semimajor axis (Fig. 3). It is evident that such a solution should contain spiral structures. In particular, the non-uniformity

of the motion along the elliptical flowline will result in increasing of density in apastron, so the curve passing through apastrons (black circles marked by  $A_1, A_2, ..., A_5$  in Fig. 3) will be a spiral density wave. The curve connected places of maximal attachment of flowlines (gray circles in Fig. 3) is the spiral density wave as well. After passing the apastron the particle's velocity increases (including the radial component of velocity  $v_r$ ) so we can expect the increasing of radial component of matter flux  $F_{rad} \propto \rho v_r$  since both  $\rho$  and  $v_r$  increase. Note, if our mechanism is consistent, the gas particle moving along the flowline should show increase of density first and after that the increase of radial velocity. Hence, the density wave should precede the pike of radial component of flux matter, and the curve passing through these peaks also will be a spiral. The increasing of radial component of flux matter after passing the wave will result in accretion rate increase in the region where "precessional" wave touches the accretor.

Resuming the qualitative consideration of changes occurring during the transition from a hot accretion disc to the cool one we can state the following:
(i) the region that is not subjected by gasdynamical perturbations appears in the cool accretion disc; (ii) the retrograde precession generates the spiral density wave in the disc's inner parts that are not subjected by gasdynamical perturbations; (iii) the rotational velocity of this wave is determined by the mean precession rate of flowlines; iv) the increase of radial component of the flux of matter takes place after gas particles pass the wave, and the curve connecting the peaks of radial flux of matter has a spiral shape.

# 3 3D gasdynamical simulation of cool accretion discs. Model and results.

In order to perform the further analysis of possible existence of a spiral wave of "precessional" type in inner regions of cool accretion discs we have conducted 3D gasdynamical simulation of the disc structure for the case when the radiative cooling decreases the gas temperature to  $\sim 10^4$  K. We used rather fine difference grid (121 × 121 × 32 gridpoints along X, Y, Z axes, correspondingly) to achieve good spatial resolution in the inner part of the disc. We parallelized our code and used supercomputer of Moscow Supercomputer Center, so we could obtain solutions with good resolution on the timescale greater than period of disc precession.

Details on our numerical model can be found in [1]. We have taken the binary with parameters of dwarf nova IP Peg and adopted the mass of accretor as  $M_1 = 1.02 M_{\odot}$ , the mass of mass-losing star as  $M_2 = 0.5 M_{\odot}$ , and binary

separation as  $A = 1.42R_{\odot}$ . The finite-difference Roe-Osher method [29,30] was used to solve the gasdynamical equations. This method was tuned for solving on multiprocessor computers. 2D decomposition of computational grid with synchronization of boundary conditions was used, so one processor operated with a "stick" of cells [31]. We conducted our simulations in corotating non-inertial Cartesian coordinate frame in the upper semi-space (due to symmetry of the problem w.r.t. the equatorial plane). We have imposed free boundary conditions on outer boundaries: constant density  $\rho = 10^{-8} \rho_{L_1}$  $(\rho_{L_1}$  – density of matter in  $L_1$  point), temperature 13600 K, and zero velocity. The stream was also specified in the form of a boundary condition: matter with specified temperature, density, and velocity was injected into a zone around  $L_1$  with radius 0.014A. Adopted parameters give the mass transfer rate equal to  $\sim 10^{-9} M_{\odot}$ /year. The accretor was adopted as a sphere of radius  $10^{-2}A$ . All matter that comes to any of cells forming the accretor was taken to fall onto the star. Computational domain was given in such a way that the disc and the stream of matter from  $L_1$  would be inside the computational domain. We adopt the solution for the model without cooling [19] as initial condition. The run of model with cooling was conducted up to the time  $\approx 10$  binary's period, the computational time on supercomputer MVS1000A of Moscow Supercomputer Center was approximately 2000 hours.

The morphology of gaseous flows in considered binary is shown in Fig. 4 and Fig. 5. Figure 4 depicts the density distribution and velocity vectors in the equatorial plane of the system for two moments of time  $t = 1.26P_{orb}$  and  $t = 2.82 P_{orb}$ . The shocks, which are formed in the disc, are seen as condensed isolines. The latter on the edge of the circumdisc halo correspond to sharp decrease of density up to the background value. It is seen that the calculated flow structure is similar to that on the sketch given in Fig. 2. We can see the dense circular disc as well as the compact circumdisc halo. The interaction of gas of the circumdisc halo with the stream generates the shock – "hot line", the latter being located outside the disc. The two-armed spiral shock wave is formed in the disc. The both arms are located in the outer part of the disc. We can also see one more spiral wave located in gasdynamically unperturbed region (see scheme in Fig. 2). Figure 5 shows the zoom of density distribution and velocity vectors in the inner part of the disc for the same moments of time as in Fig. 4. Figures 4 and 5 are presented in corotational coordinate frame (i.e. in the frame rotating with the orbital period of binary). It is seen that the two-armed spiral wave is in rest for this coordinate frame (that is rather natural for a tidal wave) but the inner spiral wave rotates. The analysis of computational results shows that the wave moves as a single whole and its velocity in the inertial frame (i.e. in the observer's frame) turns out to be equal to  $\approx -0.133$  revolution per one orbital period of the

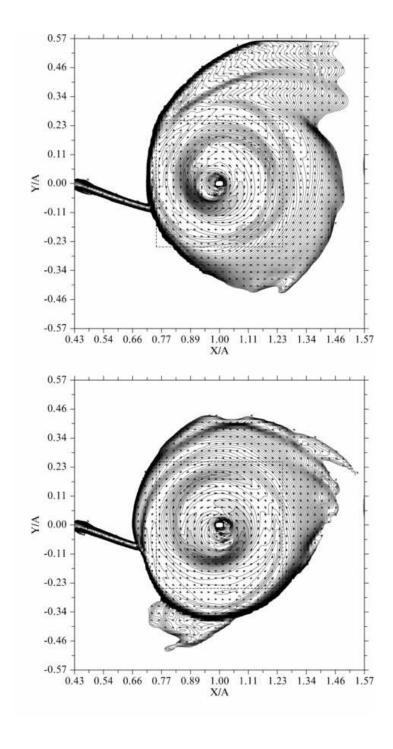


Figure 4: Density isolines and velocity vectors in the equatorial plane of binary for two moments of time  $t=1.26P_{orb}$  and  $t=2.82P_{orb}$ .

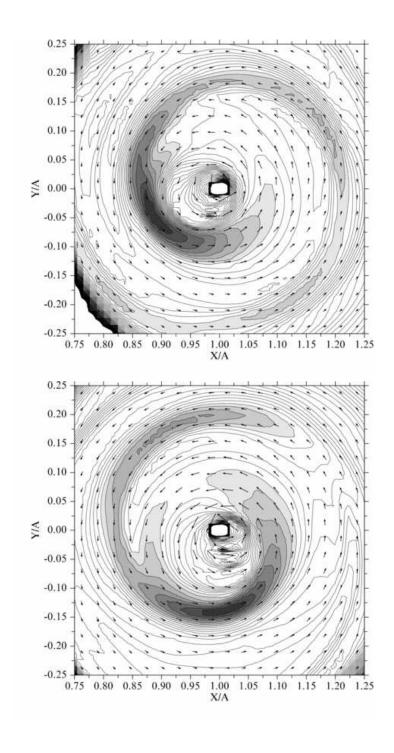


Figure 5: Density isolines and velocity vectors in the inner part of the disc (a dashed rectangular on Fig. 4) for the same moments of time as in Fig. 4.

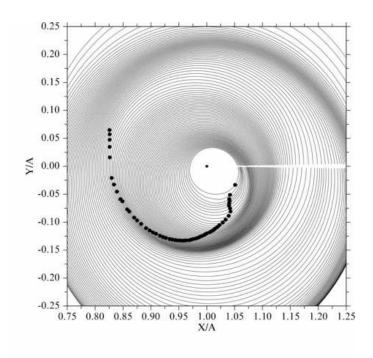


Figure 6: Calculated flowlines in the inner part of the disc for the moment of time  $t = 2.82P_{orb}$ . Apastrons for each flowline are shown by black circles.

binary. In other words, the spiral wave commits a full revolution in  $\approx 7.5$  of binary period (in inertial frame), i.e.  $P_{pr} \simeq 7.5 P_{orb}$ . The wave terminates at the distance of  $\approx 0.16A$  from the accretor. The theoretical value of period in accordance to (1) is equal to  $\approx 16 P_{orb}$ , i.e. coincides with the value computed in gasdynamical model within the accuracy of factor of two.

If our hypothesis on the precessional mechanism of generation of inner spiral wave is consistent then computed flowlines should behave in accordance with Fig. 3. The computed ones as well as the apastrons for each flowline are shown in Fig. 6. Note that the form and the position of the curve passing through apastrons coincides with those ones for density wave drawn in Fig. 5 for the same moment of time. So the computed structure is in a good agreement with our qualitative consideration. Indeed, the curve passing through apastrons constitutes the spiral density wave, the places of maximum attachment of flowlines being detached from this wave and having a spiral form as well. The distributions of density, radial velocity and radial flux<sup>2</sup> of matter along the flowline also correspond to our qualitative consideration (Fig. 7, the portion near the apastron of the flowline is shown).

<sup>&</sup>lt;sup>2</sup>per unit of square.

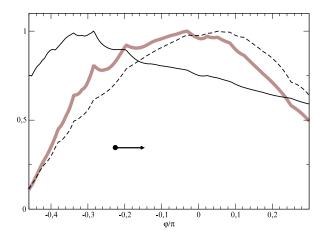


Figure 7: Distribution of density (a solid line), radial velocity (a dashed line) and radial flux of matter (bold gray line) along some flowline in the vicinity of its apastron for the same moment of time as in Fig. 6. An arrow shows the direction of motion. All distribution are normalized on its maximum values.

When moving along the flowline the density increases first (after passing the apastron) and after that the radial velocity increases, so in accordance to our forecast, the density wave precedes the pike of radial component of flux matter. Figure 8 depicts the distribution of the radial flux of matter in the equatorial plane of the disc for the time  $t = 4.92P_{orb}$  in two forms: bird-eye view and contours (the flux was normalized to its maximal value). We can see distinctly the spiralwise form of the curve passing through the flux peaks. The accretion rate increases up to the order of magnitude in comparison to the wave-free solution due to the increasing of radial flux of matter behind the "precessional" density wave.

### 4 Conclusions

The qualitative analysis of possible changes occurring in transition from a hot accretion disc to the cool one shows the possible generation of spiral wave of a new type in the accretion disc. The appearance of this spiral wave in the inner part of the disc where gasdynamical perturbations are negligible is due to retrograde precession of flowlines in the binary system.

The analysis of presented results of 3D gasdynamical simulation fully confirms our hypothesis on the possible generation of spiral wave in the inner part of the cool disc. The correspondence between the qualitative analysis and the computational results permits us to argue the precessional mech-

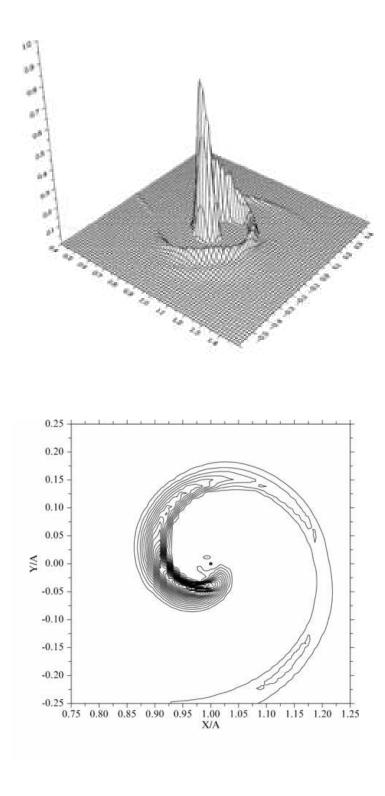


Figure 8: Distribution of radial flux of matter in the equatorial plane for the moment of time  $t=4.92P_{orb}$ . The flux is normalized on its maximum values.

anism of the wave formation. Increasing of the radial flux of matter after passing the density wave results in growth of accretion rate and formation of a compact zone of energy release on the accretor surface. This zone can be seen as a periodic increase of brightness on light curves of semidetached binaries. Observation of this zone will permit to determine the precession rate of the wave so these observations will both give the proof of the existence of "precessional" wave in the inner part of the cool accretion disc and provide the information on characteristics of the inner parts of the disc.

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